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# ИССЛЕДОВАНИЕ ДВИЖЕНИЯ И ИЗЛУЧЕНИЯ РЕЛЯТИВИСТСКИХ ЗАРЯЖЕННЫХ ЧАСТИЦ В МАГНИТНОМ ПОЛЕ В ПРИОСЕВОМ ПРИБЛИЖЕНИИ

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Исследовано движение и излучение релятивистской заряженной частицы в магнитном поле в приближении малого угла отклонения вектора скорости от оси траектории (приосевом приближении). полученные выражения для спектрально-углового распределения излучения не содержат скорости или ускорения заряженной частицы, а определены через векторный потенциал магнитного поля на прямой, в окрестности которой движется частица. Показано, что спектрально-угловое, угловое и спектральное распределения излучения содержат энергию только в виде общего множителя, произведения энергии на угол между направлением излучения и осью траектории и отношения частоты излучения к квадрату энергии, откуда следует, что форма спектрально-углового распределения излучения не зависит от энергии частицы. С изменением энергии изменяется лишь масштаб в распределении по частоте и углу между вектором направления излучения и осью траектории.

## 1. Introduction

Relativistic electron motion in different undulator magnetic field configurations have been studied by a large number of authors. The simplest model of periodic magnetic field is a one-dimensional field uniform in both transversal directions [1-7]. Such a model field is a good approximation for electrons travelling near the median plane. In the helical undulator it is necessary to take into account at least two components of the magnetic field. But in that case the field has been still approximated by the transversal components on undulator axis without al ongitudinal component [8-10]. The off-axis field may be found as a solution of the Maxwell 's equations in vacuo by using the field on the axis or in the median plane as a boundary condition. Such a method for a plane undulator has been realised in refs. [11,12]. The longitudinal component may be neglected to a first approximation. But the next order shows that the longitudinal component cause a slow betatron oscillations orthogonal to the orbit plane [11-15], with a space period of order $k^{-1}\gamma\lambda$ [11,12], where k and $\lambda$ are the undulator parameter and period respectively, $\gamma$ is the electron energy in the units of the rest mass. One can neglect the longitudinal field and the betatron motion if its oscillations period is large enough with respect to undulator length. I.e.

$$k\gamma^{-1}N << 1 \qquad (1)$$

with the number of undulator periods N.

The exact solution of equation of motion in a two - dimensional transversal magnetic field of arbitrary configuration is derived in [10]. The plane charge trajectory in a one - dimensional magnetic field is found in refs.[11,12] as a particular case in absence of longitudinal field component.

In this paper we analyse the solutions of equation of motion in the paraxial approximation in a magnetic field of arbitrary configuration. These solutions are substituted in the radiation formula, so that the spectral-angular distribution of radiation becomes a function of the magnetic potential on the trajectory axis. Two types of magnetic field are considered , namely , a field oscillating along the direction of initial velocity and a monotonously varying one. The conditions that must be hold for employment of method are pointed out. In particular, the undulator parameter k may be large enough $(k << \gamma)$.

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**2. Paraxial trajectories in magnetic field**

Let a relativistic charged particle cross a magnetic field which extends over a finite area. Refer the direction of initial velocity $\vec{v}_0$ to the x-axis. Denote by L the characteristic size of area occupied by the magnetic field along the x-axis, $H_m$ the maximum value of the magnetic field on that axis, $\vec{h}(\vec{r})$ a dimensionless function that defines the geometry of field according to $\vec{H}(\vec{r}) = H_m \vec{h}(\vec{r})$.
Let us solve the equation of motion

$$\ddot{\vec{r}} = \frac{eH_m}{m\gamma}\left[\vec{\beta}\vec{h}(\vec{r})\right] \tag{2}$$

by use of the perturbation method [16] . As a small perturbation parameter we take

$$\delta = \frac{eLH_m}{mc\gamma v_0}, \tag{3}$$

$\delta$ meaning the final deviation angle of velocity vector if the magnetic field is uniform within the distance L. In the spirit of the perturbation theory we present a particle radius-vector as an expansion

$$\vec{r}(t) = \vec{r}_0(t) + \delta\,\vec{r}_1(t) + \delta^2\vec{r}_2(t) + .. \tag{4}$$

Let $y_0$ and $z_0$ be the particle coordinates at $t \to -\infty$. The line $y = y_0, z = z_0$ we call the trajectory axis. The magnetic field can be expanded in the neighbourhood of trajectory axis as follows

$$\vec{h}(\vec{r}) = h_0 + \left(\vec{r}_\perp \vec{\nabla}_\perp\right)\vec{h}_0, \tag{5}$$

with $(\vec{r}_\perp \vec{\nabla}_\perp) = (y - y_0)\dfrac{\partial}{\partial y} + (z - z_0)\dfrac{\partial}{\partial z}$. Subscript 0 denotes the field and its derivative on the trajectory axis. Since the right part of equation (2) is proportional to $\delta$, we keep in expansion (5) only the first term of transverse coordinates.

Substituting (4) and (5) into the equation of motion (2) and equating coefficients of like powers in $\delta$ on both sides of resulting equation we find the equations for $\vec{r}_i$ *(t)*. The equation without $\delta$ gives $\ddot{\vec{r}}_0 = 0$ and therefore $\vec{r}_0 = \{v_0 t, y_0, z_0\}$.
Equating the coefficients of $\delta$ we get

$$\ddot{x}_1 = 0,\quad \ddot{y}_1 = \frac{v_0^2}{L}h_{0z},\quad \ddot{z}_1 = \frac{v_0^2}{L}h_{0y}.$$

After first integrating we find the velocity up to first order

$$\dot{\vec{r}}(t) = v_0\left[1, -\delta a_y(v_0 t), -\delta a_z(v_0 t)\right], \tag{6}$$

where the functions

$$a_y(x) = \frac{1}{L}\int_{-\infty}^{x} h_{0z}(x, y_0, z_0)dx,\quad a_z(x) = -\frac{1}{L}\int_{-\infty}^{x} h_{0y}(x, y_0, z_0)dx \tag{7}$$

are dimensionless components of the vector potential on the undulator axis. Use is made of the relation $(v_0 + \delta\dot{x}_1)^2 + \delta^2\dot{y}_1^2 + \delta^2\dot{z}_1^2 = v_0^2$, which gives in first order $\dot{x}_1 = 0$. Let us define the longitudinal coordinate up to second order keeping in the transversal components only the first order. The term of order $\delta^2$ in $v_x$ can be found from equation $\left(v_0 + \delta^2\dot{x}_2\right)^2 + \delta^2\dot{y}_1^2 + \delta^2\dot{z}_1^2 = v_0^2$ :

$$v_x(t) = v_0\left[1 - \frac{1}{2}\delta^2 a^2(v_0 t)\right], \tag{8}$$

with $a^2(x) = a_y^2(x) + a_z^2(x)$.

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A direct integration of the second-order equation following from (2) gives the same result.

Finally, after the next integrating of (6) and (8) we get the trajectory

$$x(t) = v_0 t - \frac{1}{2}\delta^2 \int_{-\infty}^{v_0 t} a^2(x)dx,$$

$$y(t) = y_0 - \delta \int_{-\infty}^{v_0 t} a_y(x)dx, \tag{9}$$

$$z(t) = z_0 - \delta \int_{-\infty}^{v_0 t} a_z(x)dx.$$

Solutions (9) are the generalisation of well known expressions for the trajectory of a charged particle in the plane sinusoidal or helical wigglers [17,18].

Now let us find out the conditions of smallness of the second order terms. The second term in (8) is much smaller as the first one when $\delta\, a(x) << 1$. If the magnetic field does not oscillate, i.e. the characteristic length of field variation is comparable with L, then the functions $a_i(x)$ in (7) have an order about unity and therefore the expansion (8) is valid if

$$\delta << 1. \tag{10}$$

The oscillating field will be considered below.

To estimate the next terms in y and z coordinates we take the second-order equations of motion following from (2)

$$\ddot{y}_2 = \frac{v_0}{L}\left\{-\frac{1}{\delta} v_0 (\vec{r}_\perp \vec{\nabla}_\perp) h_{0z} + \beta_{1z} h_{0x}\right\},$$

$$\ddot{z}_2 = \frac{v_0}{L}\left\{\frac{1}{\delta} v_0 (\vec{r}_\perp \vec{\nabla}_\perp) h_{0y} - \beta_{1y} h_{0x}\right\}. \tag{11}$$

Let us require $\delta\left|\ddot{\vec{r}}_{2\perp}\right| << \left|\ddot{\vec{r}}_{1\perp}\right|$, or

$$\left|v_0 (\vec{r}_\perp \vec{\nabla}_\perp) \vec{h}_{0\perp} - \delta \vec{v}_{1\perp} h_{0x}\right| << v_0 \left|\vec{h}_{0\perp}\right|, \tag{12}$$

where $\vec{h}_\perp = \{0, h_y, h_z\}$.

This condition will be satisfied if

$$\left|(\vec{r}_\perp \vec{\nabla}_\perp) h_{0\perp}\right| << \left|\vec{h}_{0\perp}\right|, \tag{13}$$

$$\left|\delta \vec{v}_{1\perp} h_{0x}\right| << v_0 \left|\vec{h}_{0\perp}\right|. \tag{14}$$

The relation (14) is hold when $\delta << 1$ because it follows from (6) that $\left|\vec{v}_{1\perp}\right| \sim v_0 \left|\vec{h}_{0\perp}\right|$, and $h_{0x}$ has an order of unity. As to inequality (13) it means that the magnetic field is uniform enough within a deflection of the particle. Satisfying of this conditions allows to express solutions (9) in terms of vector potential on the trajectory axis.

### 3. Motion in an oscillatory magnetic field

The condition $\delta << 1$ restricts the product of field strength and the length of trajectory in that field. By a fixed field strength it limits the length $L$. In the oscillatory field a deviation angle of velocity can be small on a much longer path than in a monotonous field. In particular, if the usual for the undulators requirement

$$\int_{-\infty}^{\infty} \vec{H}_{\perp}(x, y_o, z_o)dx = 0 \tag{15}$$

is satisfied, then under certain conditions the velocity deviation angle can be small for an arbitrary long path. It is therefore interesting to specify the conditions (10) and (12) for an oscillatory field.

Let the magnetic field vary quasiperiodically along the $x$-axis with a period $\lambda$ and the transversal component of field satisfying (15). Then the functions $a_i(x)$ in (7) have a value of order $h_{0i}\lambda / L$ and $a^2(x) \sim \frac{\lambda^2}{L^2} \cdot \left|\vec{h}_{0\perp}\right|^2$.

Requirement of smallness of the second term in (8) gives

$$\frac{\delta\lambda}{L} << 1 \text{ or } \widetilde{\delta} << 1 \tag{16}$$

where

$$\widetilde{\delta} = \frac{\lambda}{L} \cdot \delta = \frac{eH_m\lambda}{mcv_0\gamma}. \tag{17}$$

The right-hand side of equations (11) under certain relations between oscillation phases of $\vec{\beta}_{\perp}$ and $\vec{h}_{\perp}$ or $\vec{r}_{\perp}$ and the gradient of $\vec{h}_{0\perp}$ can contain a constant components which cause a slow side bend of particle or betatron oscillations. To neglect such effects one can demand the second-order terms in transversal velocity to be small with respect to the first-order ones: $\delta \cdot \left|\vec{v}_{2\perp}\right| << \left|\vec{v}_{1\perp}\right|$. Substituting (6) in (11) we get after integration

$$\left|\int_{-\infty}^{v_0 t}\left[v_0(\vec{r}_{\perp}\vec{\nabla}_{\perp})\vec{h}_{\perp} - \delta\vec{v}_{1\perp}h_{0x}\right]dx\right| << v_0 \cdot \left|\int_{-\infty}^{v_0 t}\vec{h}_{\perp}dx\right|. \tag{18}$$

For an oscillatory field satisfying (15) one can estimate

$$\left|\int_{-\infty}^{x}\vec{h}_{\perp}dx\right| \sim \lambda .$$

Then (18) can be rewritten in a form

$$\frac{1}{\lambda} \cdot \left|\int_{-\infty}^{v_0 t}\left[\left(\vec{r}_{\perp}\vec{\nabla}_{\perp}\right) \cdot \vec{h}_{\perp} - \delta \cdot \frac{\vec{v}_{1\perp}}{v_0} \cdot h_{0x}\right]dx\right| << 1 . \tag{19}$$

Relations (16) and (19) are the sufficient conditions of a validity of the trajectory eqs. (9) for an oscillatory field.

In particular, let us consider a plane undulator with, for example, $h_y = 0$. In that case $v_{1z} = \dot{z}_1 = 0$, eqs. (11) are reduced to

$$\ddot{y}_2 = \frac{v_0^2}{\delta L}\left(y - y_0\right)\frac{\partial h_{0z}}{\partial y}, \quad \ddot{z}_2 = \frac{v_0}{L}v_{1y}h_{0x}. \tag{20}$$

and (19) is fulfilled if

$$\frac{L}{y}\left(y - y_0\right)\frac{\partial h_{0z}}{\partial y} << 1, \quad \frac{\delta L}{\lambda v_0}v_{1y}h_{0x} << 1 .$$

Taking into account that $\left(y - y_0\right) \sim \frac{\delta\lambda^2}{L}$ and $v_{1y} \sim \frac{v_0\lambda}{L}$ we get

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$$\tilde{\delta}\cdot L\cdot\frac{\partial h_{0z}}{\partial y}<<1,\quad \tilde{\delta}\cdot\frac{L}{\lambda}\cdot h_{0x}<<1. \tag{21}$$

The latter unequations limit the length of plane undulator.

Eqs. (20) are the well known expressions for a defocusing force in the trajectory plane and a betatron oscillations in the xz-plane [11]. The first of conditions (21) ensures that the deviation of the particle from the trajectory axis under the defocusing force caused by the field nonuniformity is small enough. The second condition coincides with (1) since $\delta\sim k\cdot\gamma^{-1}$, $h_{0x}\sim 1$ (it should be $h_{0x}\sim h_{0z}$ to satisfy Maxwell s equations [11,12]). It demands the undulator length to be much less than the betatron motion period.

**4. Radiation**

The solutions (9) of the equation of motion gives a possibility to express a spectral and angular distribution of charge radiation in terms of the on-axis magnetic field $\vec{h}(x,y_0,z_0)$. For this sake we substitute the coordinates (9) and velocity (6) with the second order (8) in Fourier expansion of the radiation field [19]

$$E_j(\omega)=\frac{ie\omega}{cR}\cdot\int\limits_{-\infty}^{\infty}(\vec{e}_j\cdot\vec{\beta})\cdot e^{i\left(\omega t-\vec{k}\vec{r}\right)}dt,\qquad j=\sigma,\pi. \tag{22}$$

The subscript j denotes a polarisation components, $\vec{k}=\omega\cdot\vec{n}/c$ is a wave vector, $\vec{n}$ is an unit vector of the radiation direction. The unit vectors of polarisation we take as usually ($\vec{e}_z$ to be an unit vector along the $z$-axis)

$$\vec{e}_\sigma=\frac{\left[\vec{e}_z\vec{n}\right]}{\left\|\left[\vec{e}_z\vec{n}\right]\right\|},\qquad \vec{e}_\pi=\left[\vec{e}_\sigma\vec{n}\right].$$

It is convenient to transform the independent variable $t$ to the axial one $x$. Performing this transformation and expanding the $\sigma$- and $\pi$- projections of $\beta$ for small $\delta$ we get up to first order

$$\begin{gathered}\beta_\sigma=\left(\vec{e}_\sigma\vec{\beta}\right)=-\frac{\beta_0}{\sqrt{1-n_z^2}}\cdot\left[n_y+\delta n_x a_y(x)\right],\\ \beta_\pi=\left(\vec{e}_\pi\vec{\beta}\right)=\frac{\beta_0}{\sqrt{1-n_z^2}}\cdot\left[n_x n_z-\delta n_x n_z a_y(x)+\delta(1-n_z^2)a_z(x)\right].\end{gathered} \tag{23}$$

In the exponent in (22) we keep the second-order term in $v_x$:

$$f(x)=ct-(\vec{n}\vec{r})=\frac{1}{\beta_0}\cdot\left\{x(1-\beta_0 n_x)+\delta\cdot\int\limits_{-\infty}^{x}\left[\frac{\delta}{2}\cdot a^2(x)+\beta_0(\vec{n}\vec{a}(x))\right]dx\right\}, \tag{24}$$

where $\vec{a}(x)=\{0,a_y(x),a_z(x)\}$.

The term of order $\delta^2$ is retained for next reason: if the particle has an ultrarelativistic energy, then $n_y\sim n_z\sim\gamma^{-1}$ and $1-\beta_0 n_x\sim\gamma^{-2}$. Thus all the terms in (24) are of equal order with respect to $\delta$ and $\gamma^{-1}$.

The spectra-angular distribution of radiated energy is proportional to square the module of $E(\omega)$

$$\frac{d\varepsilon_j}{d\Omega\,d\omega}=\frac{e^2\omega^2}{4\pi^2 c v_0^2}\cdot\left|\int\limits_{-\infty}^{\infty}\beta_j(x)\cdot e^{i\omega f(x)/c}dx\right|^2. \tag{25}$$

The last formula can be reduced by some additional assumptions about the particle energy.

Let us consider two ranges of energy.

1. The particle energy is of order or less than its rest energy, i.e. $\gamma <\sim 1$. Then the deviation angle $\delta$ in the paraxial approximation is far less than the radiation cone $\gamma^{-1}$. The radiation has in that case a dipole nature and one may use the formula [19]

$$E(\omega) = \frac{e}{cR\left(1-\vec{\beta}_0\vec{n}\right)^2} \cdot \left[\vec{n}\left[\left(\vec{n} - \vec{\beta}\right)\dot{\vec{\beta}}(\omega')\right]\right], \text{ where } \omega' = \omega \cdot (1 - \vec{\beta}_0\vec{n})$$

A separation of $\vec{E}(\omega)$ into components of polarisation and substitution $\dot{\vec{\beta}}(\omega)$ from equation of motion yields the following formulae for spectra-angular distribution

$$\frac{d\varepsilon_\sigma}{d\Omega d\omega} = \frac{e^4\left(\beta_0 n_y\left[n_z H_y(\mathcal{H}) - n_y H_z(\mathcal{H})\right] + n_x(1 - \beta_0 n_x) H_z(\mathcal{H})\right)^2}{4\pi^2 m^2 \gamma^2 c^5\left(1 - n_z^2\right)\left(1 - \beta_0 n_x\right)^4},$$
$$\frac{d\varepsilon_\pi}{d\Omega d\omega} = \frac{e^4\left(n_z\left[n_y H_z(\mathcal{H}) - n_z H_y(\mathcal{H})\right] + (1 - \beta_0 n_x) H_y(\mathcal{H})\right)^2}{4\pi^2 m^2 \gamma^2 c^5\left(1 - n_z^2\right)\left(1 - \beta_0 n_x\right)^4}, \tag{26}$$

$$\frac{d\varepsilon}{d\Omega d\omega} = \sum_j \frac{d\varepsilon_j}{d\Omega d\omega} = \frac{e^4\left((1 - \beta_0 n_x)^2 H^2(\mathcal{H}) - \left(1 - \beta_0^2\right)\left[H_z(\mathcal{H})n_y - H_y(\mathcal{H})n_z\right]^2\right)}{4\pi^2 m^2 \gamma^2 c^5\left(1 - n_z^2\right)\left(1 - \beta_0 n_x\right)^4}, \tag{27}$$

where

$$\vec{H}(\mathcal{H}) = \int_{-\infty}^{\infty} \vec{H}(x, y_0, z_0) \cdot e^{i\mathcal{H}x} dx, \quad \mathcal{H} = \frac{\omega}{v_0}(1 - \beta_0 n_x).$$

It is obvious that expressions (26) and (27) are valid for an ultra-relativistic motion too if the main condition $\delta << \gamma^{-1}$ is satisfied.

2. The particle energy is large compared with the rest one ($\gamma >> 1$). Let us introduce a spherical coordinates $\vartheta$ and $\varphi$ as follows $\vec{n} = \{\cos\vartheta, \ \sin\vartheta \ \cos\varphi, \ \sin\vartheta \ \sin\varphi\}$. For $\gamma >> 1$ the angle $\vartheta$ is of magnitude $\gamma^{-1}$. Hence we may expand (25) with (23) and (24) for small $\gamma^{-1}$ and $\vartheta$. Then, to the first order, we get $\beta_\sigma = -\vartheta \cdot \cos\varphi - \delta \cdot a_y(x)$, $\beta_\pi = \vartheta \cdot \sin\varphi + \delta \cdot a_z(x)$.

All items in function $f(x)$ turn out to be small quantities of second order

$$f(x) = \tfrac{1}{2} \cdot \gamma^{-2} \cdot \int_{-\infty}^{x}\left\{1 + \left[\vec{p} + \delta \cdot \vec{a}(x)\right]^2\right\}dx, \tag{28}$$

with $\vec{p} = \{0, \psi\cos\varphi, \psi\sin\varphi\}$, $\quad \psi = \vartheta \cdot \gamma$.

Substituting from these approximations in equation (25) we get

$$\frac{d\varepsilon_{\sigma,\pi}}{d\Omega d\omega} = \frac{e^2\omega^2}{4\pi^2\gamma^2 c^3}\left|\int_{-\infty}^{\infty}\left[\vec{p} + \delta \cdot \vec{a}(x)\right]_{y,z} e^{i\omega f(x)/c} dx\right|^2. \tag{29}$$

## 5. Conclusions

The equation (25) with (23) and (24) express the spectra-angular distribution of radiated energy in terms of $\vec{a}(x)$, reduced potential of magnetic field on the trajectory axis. The validity conditions of paraxial approximation are given by (10) and (13) for non-oscillatory field and by (16) and (19) for oscillatory one. These conditions mean that the deviation of velocity vector from its initial direction is

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small and the magnetic field in the vicinity of the trajectory is homogeneous enough. In case of oscillating field the dimensions of the region in which the magnetic field is non-zero are to be far less then the betatron oscillations period. The general expression for the radiation distribution allows a simplification in a wide range of energy value, namely $\gamma <\sim 1$ and $\gamma >> 1$.

As is well known, the radiation pattern depends essentially on the value of k, undulator parameter. It differs from product $\gamma \cdot \widetilde{\delta}$ by a factor about of unity. The main condition of paraxial approximation demands $k << \gamma$. Hence in the case of ultrarelativistic motion the formula (29) is applicable for any $k$ practically used in undulators.

Notice that the ultrarelativistic formula (29) includes the particle energy only as a scale factor in product $\psi = \gamma \cdot \vartheta$ and $\gamma^{-2}\omega$ in the exponent. Therefore the shape of the spectra and angular distribution in considered approximation does not depend on the particle energy. The energy variation changes only the scale of the frequency, the angle $\vartheta$ and the magnitude of the energy emitted. This was pointed out in early papers dealt with an ultrarelativistic undulator radiation [4,5]. Here we establish it for the radiation in an arbitrary magnetic field satisfying the considered conditions.

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# Paraxial approximation for the charge radiation in a magnetic field

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A paraxial method for calculation of the relativistic charge radiation in a magnetic field is presented. The method is based on expanding of motion and radiation formulae for the small angle between the velocity vector and the trajectory axis. Some other conditions that allow to apply the method are determined. The obtained expressions for the spectral and angular distribution of radiation does not contain the velocity or acceleration of charge: they are expressed through the vector potential on a trajectory axis. It is shown that the radiation formulae for ultrarelativistic charge include the charge energy only as scale factors by the frequency, the angle between the trajectory axis and direction of radiation and by the radiation intensity. Hence the shape of angular and spectral distribution does not depend on the electron energy.

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